E-M I, PHYS301, Department of Physics, University of Crete

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I. MAXWELL'S EQUATIONS: A THEORY OF FIELDS

A. Electric and Magnetic Field: the Output of Maxwell's equations

Electromagnetic phenomena are mediated by forces caused by charges and their motion. The fundamental problem that we deal with in this course can be defined as follows. Consider a number of point particles with charges $Q_1, \cdots Q_N$, where N can be very large ($N \to \infty$), located at positions or trajectories $\mathbf{r}_i$(t), i=1,$\cdots$N. What is the electromagnetic force (i.e. absent if these same particles have Q_i=0) that these charges exert on a "test" charge q located at a given position and moving with a given velocity?

There are two ways of viewing electromagnetic phenomena. The older one thinks in terms of forces between charges or currents that result from "action at a distance". Coulomb's law of electrostatics and the corresponding law of magnetostatics were first stated in this way. Two charges interact with a force determined by their positions, velocities, and accelerations. Faraday introduced a new point of view, which formulates electromagnetism as a "field theory". One envisions the entire space as filled with a field created by the charges $Q_1, \cdots Q_N$, whose existence is felt via a force exerted on a "test" charge q. Faradays approach led to Maxwells field equations, which brought to bear on the electromagnetic problem the well developed theory of continuum mechanics and elasticity. From Faradays point of view, we can describe the effects of $Q_1, \cdots Q_N$ by defining two vector functions, $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$, at each and every position $\mathbf{r} = (x, y, z)$. These two fields are inter–dependent in a time–dependent physical system and together define the electromagnetic field. They are present at point $\mathbf{r}$ even when there is no charge q there, as long as $Q_1, \cdots Q_N \neq 0$. However, the prexisting fields become observable *only* when q is introduced, through the force that it feels.

What *is* an electromagnetic field? Mathematically, it is defined in terms of the force $\mathbf{F}(\mathbf{r}, t)$ exerted on a "test" charge q introduced at time t at position $\mathbf{r}$ with velocity $\mathbf{v}$. Experiment showed that q feels a force consisting of two parts: one independent of $\mathbf{v}$, called the *electric* *force*, and one proportional to $\mathbf{v}$ and orthogonal to its direction, called the *magnetic force.* The total force can be expressed in terms of two vector functions that characterize the field at every point $\mathbf{r}$ in space at time t: the *electric field* $\mathbf{E}(\mathbf{r}, t)$ and the *magnetic field* $\mathbf{B}(\mathbf{r}, t)$. These vector functions are defined mathematically by Lorentz's equation. This equation tells us that the force $\mathbf{F}(\mathbf{r}, t)$ felt by a *point* charge q located at position $\mathbf{r}$ at time t while moving with velocity $\mathbf{v}$ can be expressed as follows:

$$\mathbf{F}(\mathbf{r}, t) = q[\mathbf{E}(\mathbf{r}, t) + \mathbf{v} \times \mathbf{B}(\mathbf{r}, t)].$$

We note that, in general, the electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ are affected by the charge q that senses them. For example, q can change the positions of $Q_1, \cdots Q_N$ that caused the field via the force that it exerts on them. We thus *define* the electric field caused by a distribution of charges $Q_1, \cdots Q_N$ by taking the limit $q \to 0$ so that q (with $\mathbf{v}$=0) does not disturb $Q_1, \cdots Q_N$:

$$\mathbf{E}(\mathbf{r}, t) = \frac{\mathbf{F}(\mathbf{r}, t)}{q} \ , \ q \to 0$$

It is verified experimentally that the above ratio $\mathbf{F}/q$ is independent of q. We then define the magnetic field vector as follows:

$$\mathbf{v} \times \mathbf{B}(\mathbf{r}, t) = \frac{\mathbf{F}(\mathbf{r}, t)}{q} - \mathbf{E}(\mathbf{r}, t) \ , \ q \to 0.$$

OK, we defined the fields, but what *are* they really? Electric and magnetic fields are abstract quantities that are very hard to visualize or grasp conceptually. Nobody has actually "seen" the fields, but everybody has felt their presence. We can loosely say that the introduction of electric charges $Q_1, \cdots, Q_N$ fills the entire space by an electromagnetic field. We can verify that the universe is indeed "distorted" at position $\mathbf{r}$ by the presence of $Q_1, \cdots, Q_N$ by observing that a force is now exerted when an object with charge q is placed at $\mathbf{r}$. This force is zero if instead we place an object with $q = 0$. For example, we only "see" the electric and magnetic field that fills this room when we turn on our cellular phone. The field is there even if our phone is turned off, we just don't see. To make an analogy, just like a propagating wave disturbance is created when we distort a medium at some position, e.g. when we throw a stone in the water, the introduction of an object with electric charge changes its surrounding space by creating an electromagnetic field that eventually fills the entire space. To describe a wave disturbance, we need to know the time–dependent changes in some property of the medium from its equilibrium values. For example, to describe a "wave" in a football stadium, we need to know the displacement $\Psi(\mathbf{r}, t)$ of the spectator sitting on chair $\mathbf{r}$ from his/her equilibrium position (the chair) at every time t and for every chair $\mathbf{r}$. In an analogous way, the "displacement" at point $\mathbf{r}$ caused by electric charge is described by the electric and magnetic field vector functions $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$. To make an analogy with gravity, we can think of the Earth, of mass M, as changing the properties of its surrounding space by "filling" it with a gravitational field. The latter field is described by the acceleration of gravity, whose magnitude at a distance r from the Earth's center is given by $g(r) = GM/r^2$. The gravitational field $g(r)$ is similar to the electric field $E(r)$ (recall Coulomb's force). Although it is hard to visualize or understand what the gravitational field really is, we accept that it is present when an object of mass m placed at position r experiences a force $mg(r)$. In an analogous way, we know that an electric field exists at point $\mathbf{r}$ if an object of charge q experiences a force of $q\mathbf{E}(\mathbf{r})$ when placed there.

The concept of fields allows us to describe forces between objects that do not touch each other, using a "field theory" rather than a "force theory". We note here that, if all we were interested in was to describe forces between isolated charges that do not move, we could talk in terms of forces without invoking fields. The field theory point of view does not offer a significant practical advantage in this case. However, when it comes to a moving charge Q which "transfers its motion" to another charge q far away after a finite time has elapsed, it is advantageous to talk in terms of an electric field, due to the motion of Q, that travels at the speed of light from point to point until it reaches q. The concept of instantaneous forces violates causality and thus breaks down in this case.

B. Microscopic Maxwell's Differential Equations

Maxwell's equations are the differential equations whose solution determines the electric and magnetic fields caused by a given charge and current distribution in space. These distributions are described by two functions, $\rho(\mathbf{r}, t)$ (the *charge density*) and $\mathbf{J}(\mathbf{r}, t)$ (the *current density*), whose definition and physical meaning is discussed below. Maxwell's equations are the following coupled differential equations, which must be solved for $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ given $\rho(\mathbf{r}, t)$ and $\mathbf{J}(\mathbf{r}, t)$ (pages 47–54, Griffiths Vol. 2):

$$\nabla \cdot \mathbf{E}(\mathbf{r}, t) = \frac{\rho(\mathbf{r}, t)}{\epsilon_0} \quad (Gauss's\ Law)$$

$$\nabla\times\mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \quad (Faraday's\ Law)$$

$$\nabla\cdot\mathbf{B}(\mathbf{r},t) = 0$$

$$\nabla\times\mathbf{B}(\mathbf{r},\mathbf{t}) = \mu_0\mathbf{J}(\mathbf{r},t) + \epsilon_0\mu_0\frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t} \quad (Ampere - Maxwell\ Law),$$

where we used the SI system of units. $\epsilon_0 = 8.85\times10^{-12}\frac{C^2}{Nm^2}$ is called the dielectric constant of the vacuum and $\mu_0 = 4\pi10^{-7}N/A^2$ is called the permeability of the vacuum (free space). Maxwell's differential equations relate the field with its sources at the same position $\mathbf{r}$. Note that Maxwell's contribution to the above equations, other than writing them together in a compact and intuitive notation (very important for making progress since it facilitates the mathematical manipulations), was the last term $\epsilon_0\mu_0\frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t}$. The importance of this term is discussed in pages 49–51 of Griffiths Vol. 2. Use your calculator to obtain the numerical value of $1/\sqrt{\epsilon_0\mu_0}$, a quantity with units of m/s, using the above values of the constants ϵ_0 and μ_0 that had been obtained experimentally prior to Maxwell. The answer is 300000km/s, which just happened to coincide with the known speed of light. Accident? I don't think so.

The above expressions are a compact way of writing down eight coupled differential equations with six unknown functions of four variables each, $E_x(x,y,z,t)$, $E_y(x,y,z,t)$, $E_z(x,y,z,t)$, $B_x(x,y,z,t)$, $B_y(x,y,z,t)$, $B_z(x,y,z,t)$, in a way that applies to any coordinate system. This is made possible by introducing a vector operator ∇ defined as (Griffiths Vol 1 pages 26–37)

$$\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}),$$

which behaves like a vector. It is clear that we are dealing with a difficult mathematical problem, so at least we have a compact formalism that allows us to perform mathematical operations in an intuitive way, by drawing from our knowledge and intuition about vector algebra. We have for example that

$$\nabla\cdot\mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

just like the usual dot product, while $\nabla\times\mathbf{E}$ is calculated as any cross product of two vectors:

$$\begin{aligned}
\nabla\times\mathbf{E}\big|_x &= \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\\
\nabla\times\mathbf{E}\big|_y &= \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\\
\nabla\times\mathbf{E}\big|_z &= \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}
\end{aligned} \tag{1}$$

Be careful however:

$$\nabla \cdot \mathbf{E} \neq \mathbf{E} \cdot \nabla$$

unlike for normal vectors where ordering does not matter for the inner product. In all cases, the derivatives act on *everything* that comes after them. Importantly, we can use the vector derivative expressions found in the cover of Griffith's book to transform Maxwell's equations in different coordinate systems and simplify their solution by taking advantage of possible symmetries of the physical system at hand, as discussed later. More mathematical details may be found in Griffith's book, pages 26–37 and in the mathematical formulas in the beginning of the book. Note that, just like any other differential equation, to solve Maxwell's system of differential equations you need to also have some *boundary conditions*.

A very important mathematical property, extremely useful in practice, is the *uniqueness* of the differential equation solution for given boundary conditions. What this means is that if we are able to find, *in whatever way*, two vector functions $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ whose derivatives satisfy the above differential equations for given $\mathbf{J}(\mathbf{r}, t)$ and $\rho(\mathbf{r}, t)$ and which, in addition, satisfy the given boundary conditions (i.e. have known values or space derivatives at every point on surfaces that separate different regions of space or at infinity, while their time–dependence respects causality), then these functions describe the *one and only* possible electromagnetic field. *Our goal is to find this solution in the easiest way possible.* We do not care if we derive the functions $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ in some mathematical way, obtain them numerically using the computer, or simply guess what they are in an intuitive or even counter-intuitive way. As long as we find two functions that satisfy the two above requirements we are done, no questions asked. In this course we will discuss a number of systematic ways of thinking about this problem that can guide us to find the solution by taking advantage of our physical intuition.

C. Charge and Current densities: the input to Maxwell's equations

Charge is a property of matter. It governs the strength of the electromagnetic force, just like "gravitational mass" governs the gravitational force. The more charge an object has the stronger the electromagnetic field that it creates. But what *is* charge? At the most fundamental level, we don't really know. Mechanics never told us what mass "really is", only how it behaves. We are familiar with mass simply because we have spent our lives pushing objects around. Similarly, classical electromagnetism and Maxwell's equations tell us how charge behaves, not what it is.

1. *Microscopic definition*

We describe the charge and current that create the electromagnetic field via two functions $\rho(\mathbf{r},t)$ and $\mathbf{J}(\mathbf{r},t)$, defined at every point $\mathbf{r}$ in space independent of whether there is a charge there or not. ρ and $\mathbf{J}$ act as source terms in Maxwell's equations and are defined as follows. Consider N point charges Q_i (electrons, protons,$\cdots$) that move in trajectories $\mathbf{r}_i(\mathrm{t})$. Then,

$$\rho(\mathbf{r},t) = \sum_{i=1}^{N} Q_i \delta(\mathbf{r} - \mathbf{r}_i(t)) \tag{2}$$

and

$$\mathbf{J}(\mathbf{r},t) = \sum_{i=1}^{N} Q_i \frac{d\mathbf{r}_i(t)}{dt} \delta(\mathbf{r} - \mathbf{r}_i(t)), \tag{3}$$

where

$$\delta(\mathbf{r}) = \delta(x)\delta(y)\delta(z)$$

is the three–dimensional and $\delta(x)$ the one–dimensional Dirac delta–functions, discussed in pages 62–67 of Griffith's book. The delta function is *defined* by the property

$$f(\mathbf{r}) = \int d\mathbf{r}' f(\mathbf{r}')\delta(\mathbf{r} - \mathbf{r}') \tag{4}$$

for *any* function $f(\mathbf{r})$, where $d\mathbf{r}'{=}dx'dy'dz'$. Formally, it is not a regular function and does not have meaning unless it enters in an integral as above. $\delta(x{-}x_0)$ can however be approximated by the limit of a function that vanishes everywhere except for $x{=}x_0$, where it becomes infinite, while $\int dx\delta(x){=}1$ (finite area). For more details, see Griffiths.

2. *Macroscopic approximation: Charge Density*

$\rho(\mathbf{r})$ and $\mathbf{J}(\mathbf{r})$ defined as above become *infinite* at the positions $\mathbf{r}{=}\mathbf{r_i}(\mathrm{t})$ occupied by the N particles at the time of interest, while they *vanish* at all points $\mathbf{r}$ in space where there is no charge present. Therefore, they exhibit a wild variation as function of $\mathbf{r}$ on a length scale comparable to atomic distances. However, the electromagnetic fields that we feel in everyday life are smooth functions of $\mathbf{r}$ on the macroscopic lengthscales of interest.

Furthermore, the practical use of the above definitions is limited since it is impossible to know the exact trajectories of $N \to \infty$ point particles. Fortunately, we can approximate ρ and $\mathbf{J}$ by smoothly varying functions defined below. This turns out ot be an excellent approximation at macroscopic lengthscales, i.e. for distances much larger than atomic dimensions, and we will use these smooth functions as input to Maxwell's equations.

We start with $\rho(\mathbf{r})$. In practice, we are interested in describing electromagnetic fields produced by aggregates of a very large (macroscopic) number of $N\to\infty$ elementary charges (electrons and protons). We note from the definition of the Dirac delta–function that

$$\int_V d\mathbf{r}'\rho(\mathbf{r}') = Q_{in} = \sum_{i=1}^{N} Q_i$$

gives the total charge Q_{in} located inside the volume V. Let us consider a volume $V{=}\Delta V$ centered at point $\mathbf{r}$. We take the dimensions of this ΔV to be *much smaller* than the dimensions and characteristic lengths of the physical system we are studying, but at the same time *much larger* than atomic distances, so that ΔV contains a macroscopic number, practically $N \to \infty$, of point particles. If we calculate the electromagnetic field by approximating the microscopic $\rho(\mathbf{r})$, Equation (2), by its average value within the small volume ΔV around $\mathbf{r}$, there will be no observable difference from the electromagnetic field calculated by substituting Equation (2) in Maxwell's equations, provided that the distances $\mathbf{r}$ we are interested in are larger than the size of ΔV defined as above. For example, the electromagnetic field produced by a small piece of paper is zero at all distances of practical interest to us, which are much larger than atomic sizes, because the average charge within ΔV is zero if the atoms are neutral. The answer would be very different however if we measured the electromagnetic field at length scales within the atoms. But this is not necessary for our purposes. We thus substitute in Maxwell's equations the average charge density defined as

$$\rho(\mathbf{r}, t) \to \frac{1}{\Delta V}\int_{\Delta V} d\mathbf{r}'\rho(\mathbf{r}', t) = \frac{\sum_{i=1}^{N} Q_i}{\Delta V} = \frac{\Delta Q(t)}{\Delta V},$$

where $\Delta Q{=}\sum_{i=1}^{N} Q_i$ is the total charge within $\Delta V \to 0$, obtained from Equation (2) and the delta–function definition Equation (4). Corrections to this approximation will be considered later in this course when we deal with polarization. In this way, Maxwells theory deals with continuous charge distributions in a way analogous to the theory of elasticity and continuum mechanics. The graininess (discreteness) of the charge distribution is ignored in such a macroscopic treatment, an excellent approximation at the right length scales. We average out the microscopic charges within a sufficiently small volume around each position $\mathbf{r}$ of interest,

$$\Delta V \to dV = d\mathbf{r} = dxdydz = rdrd\phi dz = r^2 \sin\theta drd\theta d\phi. \quad (5)$$

We can consider $\Delta V \to 0$ and use the above mathematical expression for the infinitesimal volume element if ΔV is smaller than the characteristic lengths and large enough to contain a macroscopic number N of point charges. Equation (5) expresses ΔV in terms of cartesian, cylindrical, and spherical coordinates respectively (see Griffiths pages 53–60).

A further simplification is used if the charge of interest is contained within a volume $\Delta V = \Delta S\delta$ of very small thickness δ in the direction perpendicular to the surface ΔS. If δ is much smaller than the characteristic lengths of interest, we can ignore the details of the rapid variation of the electromagnetic fields within δ and consider the charge as restricted within an ideal two–dimensional surface with thickness $\delta \to 0$, rather than a three–dimensional volume with negligible thickness. In this limit, ρ becomes very large:

$$\rho = \frac{dq}{dV} = \frac{dq}{\delta dS} \to \infty$$

when $\delta \to 0$. the 3D volume reduces to a 2D surface. It is then convenient to introduce the *surface charge density* defined as

$$\sigma(\mathbf{r} = \rho\delta = \frac{dq}{dS},$$

where $\rho \to \infty$, $\delta \to 0$, and ignore the details of the rapid change in the fields within δ by considering discontinuous fields determined by boundary conditions on the surface, as discussed later. In this case, dq is the total charge located within a surface area dS located around position $\mathbf{r}$ somewhere within the surface. Similar arguments hold if the charge is restricted along a one–dimensional system, i.e. a wire with diameter much smaller than the characteristic lengths of interest. We then describe the charge distribution by introducing the *linear charge density*

$$\lambda(\mathbf{r}) = \frac{dq}{dl},$$

where dq is the charge found within a length dl around a certain position $\mathbf{r}$ along the wire.

3. *Macroscopic approximation: Current Density*

Similar to the above–defined averaged (macroscopic) charge density ρ, we define a macroscopic current density $\mathbf{J}$ due to charge motion leading to electric current. We average out

Equation (3) and substitute in Maxwell's equations

$$\mathbf{J}(\mathbf{r},t) \to \frac{1}{\Delta V}\int_{\Delta V} d\mathbf{r}'\mathbf{J}(\mathbf{r}',t) = \frac{\sum_{i=1}^{N} Q_i \frac{d\mathbf{r}_i}{dt}}{\Delta V} = \frac{\sum_{i=1}^{N}\mathbf{J}_i}{\Delta V}$$

where Q_i, i=1,$\cdots N$ are the charges enclosed within ΔV around $\mathbf{r}$, with size as discussed in the previous section, and $\mathbf{J}_i = Q_i\mathbf{v}_i$ is the current density due to the point charge Q_i. Note that a positive charge moving with velocity $\mathbf{v}$ contributes the same as a negative charge moving with velocity $-\mathbf{v}$. To understand the physical significance of $\mathbf{J}$, we note in analogy to fluid flow that

$$I(\mathbf{r},t) = \mathbf{J}(\mathbf{r},t)\cdot d\mathbf{S} = \frac{\sum_{i=1}^{N} Q_i\Delta\mathbf{r}_i \cdot d\mathbf{S}}{\Delta t\Delta V} = \frac{\Delta Q(t)}{\Delta t}$$

is the current (charge per unit time) that crosses the surface dS located at position $\mathbf{r}$ during the time interval (t,t+Δt), flowing in the direction along the unit vector $\hat{n}$ perpendicular to dS. We defined $d\mathbf{S}$=$dS\hat{n}$. The most important result of this subsection is that we can obtain the total current that flows through a surface S by calculating the surface integral

$$I = \int_S \mathbf{J}\cdot d\mathbf{S}.$$

A material may have a bunch of differently charged species that move simultaneously, e.g. oppositely charged ions labelled by the index k, with charge densities ρ_k, each moving with average velocity $\mathbf{v}_k$. In this case, the total current is

$$\mathbf{J} = \sum_k \rho_k\mathbf{v}_k.$$

For example, in a neutral metal wire, there are both positive (immobile ions, charge density ρ_+) and negative (moving electrons, charge density ρ_-) charges. The total charge (and thus electric field) is zero: $\rho = \rho_+ + \rho_- = 0$. Despite the charge neutrality however, the current density (and magnetic field) is nonzero:

$$\mathbf{J} = \rho_+ \times 0 + \rho_-\mathbf{v} = \rho_-\mathbf{v}$$

Note that it is wrong to believe that all the charges of the same species k move with the same exact velocity $\mathbf{v}_k$; there is an extremely large number of them (e.g. current of 1A=1 C/s means $1/1.6\times 10^{19} \sim 6{\times}10^{18}$ electrons per sec). $\mathbf{v}$ entering in the current equations is the *average* velocity of all point charges of a certain species.

If the flow of charge occurs parallel to a surface, i.e. inside a volume with very small width δ, then J is very large: formally, $\delta \to 0$, $J \to \infty$. We then define the **surface current density**

$$K = J\delta = \frac{\Delta I}{\Delta l_n}$$

where $\Delta S_n = \delta \Delta l_n$ and Δl_n is the length perpendicular to the surface current.

D. Charge conservation

From the above we can derive a relation that expresses mathematically the principle of charge conservation. Consider a closed volume V with surface S. The total charge inside this volume, Q_{in}, is given by

$$Q_{in} = \int_V \rho dV$$

At the same time, the rate of charge flow across the surface that encloses the above volume is given by

$$I = \int_S \mathbf{J} \cdot d\mathbf{S} = \frac{dQ}{dt},$$

where Q is the charge that flows out of the surface. Charge conservation implies that $dQ_{in} = -dQ$ and therefore

$$\int_S \mathbf{J} \cdot d\mathbf{S} = -\frac{d}{dt} \int_V \rho dV$$

from which we deduce that

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}, \tag{6}$$

where we use a partial derivative since ρ also depends on space. The last relation stems from Gauss's theorem

$$\int_S \mathbf{J} \cdot d\mathbf{S} = \int_V \nabla \cdot \mathbf{J} dV$$

when applied for an arbitrary small volume around a certain point in space. Equation (6) is known as the *continuity equation* (note the analogies to fluid motion).

Note that, in practice, you can apply charge conservation by invoking common sense and counting correctly all charges that flows in and out of your chosen volume, using the definitions of ρ, σ, λ, I, $\mathbf{K}$, and $\mathbf{J}$ (see the solved examples).

E. Conclusions

Maxwell's equations + Lorentz force are in principle almost all that you need to know in order to predict and explain any electromagnetic phenomenon. You can feel secure with the thought that this is all. These equations are *postulates*, similar to Newton's second law. Their only real justification is that these are the minimal set of equations able to describe all electromagnetic phenomena known so far. One should note however that, in the general case, we are talking about solving a system of eight coupled differential equations to determine six unknown functions (E_x, E_y, E_z, B_x, B_y, B_z) of four variables each (x, y, z, and t). This is no easy task, and drawing conclusions appears to be pretty tough at first glance. The problem is even more complicated since the fields change $\rho(\mathbf{r}, t)$ and $\mathbf{J}(\mathbf{r}, t)$, which then change the fields etc. We don't even know the charge and current distributions, the input in Maxwell's equations! The purpose of this course is to develop some general principles that allow us to extract sufficient information from the differential equations in order to describe and predict electromagnetic phenomena in a reliable way: not perfect, just adequate for the purpose of answering the particular questions that we are asking. After all, "Perfect is the enemy of the Good".