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(Dated: February 16, 2011)

I. WHY DO WE CARE ABOUT ELECTROMAGNETISM?

What force holds the atoms of a molecule or a solid together? What force keeps your car on the road as you round a turn? What force accelerates the electrons that paint the picture on your TV or computer screen? What force underlies the beauty of a thunderstorm? With the exception of gravity, all forces that we encounter in everyday life are manifestations of a *single* force: the *electromagnetic force*. For example, the friction, tension, spring, and normal forces that we studied in mechanics are ultimately electromagnetic forces.

Electromagnetism is one of the four fundamental forces of Nature (the others are gravity, weak, and strong interaction). It governs the behavior of matter starting from the atomic scale outside the nucleus and extending into the macroscopic world. The electromagnetic force is responsible for the structure of matter. Much of physics, all of chemistry, and most of biology deal in this realm. The functions that make our bodies work rely heavily on electromagnetic forces. Life itself and the DNA replicating mechanism at its heart are manifestations of electromagnetism. Essential to our society is electromagnetic technology, ranging from computers to radios to TVs to power lines to electric lights and batteries to electric motors and generators to radiation etc. Electronic technology has led to novel devices for storing and processing information, such as the transistor, memory chips, and the computer's microprocessor. Electromagnetic waves are essential for many technological applications, but require theoretical modelling using the basic principles of electromagnetism in order to be developed. For example, the active research field of photonics relies on Maxwell's equations in order to learn how to make devices that can harness light. Classical electromagnetism combined with quantum mechanics is also an active research field of condensed matter physics and nanoscience. The principles of electromagnetism are required in order to develop and understand electrical machines, waveguides and modern telecommunication systems, medical applications, antennas and waveguides, optical fibers, plasmas, circuits and networks, wireless internet, and many more applications. Many devices with

new functionalities are yet to be discovered.

The fundamental aspects of such applications are described by Maxwell's equations. These equations are a major intellectual achievement that should be familiar to every student. To make practical use of Maxwell's equations however, it is necessary to master the art of making approximations based on the properties and dimensions of a system and on the time scales (frequencies) of importance. Despite being a theory whose basic principles are well understood, the challenge today is to use the fundamental equations, principles, and concepts in order to predict and understand behaviors of matter and/or to design new devices with desirable properties that do not exist unless we figure out how to make them. Although everything was discovered more than a century ago, so in that sense electromagnetism is an old field, simply knowing the "correct" fundamental concepts and equations is far from adequate: you have to know how to use this knowledge to predict, decide, and influence.

From a fundamental point of view, electromagnetism and the way that we will approach it is a prototype *theory of fields*. Field theory provides the framework for pushing the frontiers of modern physics. This course offers your first opportunity to discuss some of its elementary concepts and ideas. This course is not just about learning new information, it is about developing a new way of thinking and using it to get useful results for understanding and influencing real phenomena.

Our approach to the subject starts directly from the general theory, in particular from the axioms (Maxwell's equations), and derives the rest as special cases (e.g. derives Coulomb's law from Maxwell's equation). This is in contrast to the more traditional approach to teaching this subject, which follows the historical route of starting from the special cases and then building the general theory. There are advantages and disadvantages to both of these approaches. Our way should make you feel secure from day one: you know everything there is to know after the first few lectures. When taught this way, this is really a course on the theory of fields, rather than a course about forces, as is the case for elementary classical mechanics courses starting from Newton's law or for the approach to electromagnetism that starts from Coulomb's law. Fields are abstract concepts that are difficult to grasp fundamentally. Nevertheless, fields rather than forces are the basis of modern physics. To overcome the mathematical and psychological difficulties that some students develop after looking at Maxwell's equations, we will do a lot of examples on how things work in practice

and derive the special from the general, rather than the other way round. One of our main goals in this course is to learn how to *think* in terms of fields. This is not just about language. It is about a new way of thinking and approaching Physics that differs from that developed in your previous courses. It is only by thinking in this way that Physics can make progress beyond today's frontiers. My lectures will be more about how fundamental concepts work in practice, something of a "user's manual" of Maxwell's equations. What is presented as theory in the more traditional way of teaching this course will be an application or a solved example for us.

It is very important to realize that, with the exception of very few special cases (mostly symmetric systems), *the general equations cannot be solved easily*, except perhaps by using sophisticated numerical methods, including commercially available software packages, or for simplified geometries. Nevertheless, if we use the general concepts and way of thinking developed here, we can make significant progress: we can draw practical conclusions and see them work in the laboratory or in real life applications. We do not aim at getting everything, we only shoot at getting what is *adequate* for our particular needs and purposes. "Perfect is the enemy of the good". For example, a major simplification that works for some systems and situations is to expand the time dependence, starting from the static/time-independent limit, and focus only on the space dependence (e.g. in some cases we can use the frequency as an expansion parameter). Also, in many cases, the details of the sources of the electromagnetic fields are not known. We must then devise tricks to extract results from the limited information that we have, using for example boundary conditions. Not only are our mathematical and computational resources often inadequate to study the complicated realistic systems, but quite often we do not even know the input (the sources) that enter in Maxwell's equations. We must thus think critically before and after we compute. It is wrong to believe that we can understand and describe how Nature works simply by doing mathematics. Coming up with the right approximations and models is a major part of succeeding in Physics.

Without further delay, we present next the full-blown Maxwell's differential and integral equations in full complex mathematical notation. We explain and define the mathematical symbols and physical quantities that enter in these equations. We then use the rest of the course to develop the "User's Manual" for these equations.